AD-777 916

AN IMPLICIT SIDE-FIRING GUNSHIP FIRE CONTROL SYSTEM

Tom J. Forster

Air Force Academy
Colorado

February 1974

Editorial Review by Lt Colonel W. A. Belford, Jr.
Department of English and Fine Arts
USAF Academy, Colorado 80840

ACCESSION for

NTIS	White Section	☑
D.C	Buff Section	☐
UNANNOUNCED		☐
JUSTIFICATION		

BY ...
DISTRIBUTION/AVAILABILITY CODES

Dist.	AVAIL. and/or SPECIAL	
A		

This research report is presented as a competent treatment of the subject, worthy of publication. The United States Air Force Academy vouches for the quality of the research, without necessarily endorsing the opinions and conclusions of the author.

This report has been cleared for open publication and/or public release by the appropriate Office of Information in accordance with AFR 190-17 and DODD 5230.9. There is no objection to unlimited distribution of this report to the public at large, or by DDC to the National Technical Information Service.

This research report has been reviewed and is approved for publication.

Philip J. Erdle
PHILIP J. ERDLE, Colonel, USAF
Vice Dean of the Faculty

Additional copies of this document are available through the National Technical Information Service, U. S. Department of Commerce, 5285 Port Royal Road, Springfield, VA 22151.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD 777916

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) An Implicit Side-Firing Gunship Fire Control System		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Tom J. Forster, Lt Col, USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Aeronautics USAF Academy, Colorado 80840		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Dean of the Faculty USAF Academy, Colorado 80840		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this document is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Side-Firing
Gunship
Orbit Stability
Air/Ground Ballistics
Gunship Trajectory in a Wind

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The problem of operating a side-firing gunship in a closed orbit about a target in the presence of wind is investigated. The orbit is determined to be elliptical with the target at one of the foci. Orbit stability is defined and two families of stable orbits for various ratios of wind speed to airspeed are calculated from the criterion for stability. The ballistic equations for side-firing guns (drag-dominated trajectory with gravitational perturbation) are solved analytically to determine ballistic drop. Ballistic-orbit matching

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20.
is accomplished to determine the criterion for matching muzzle velocity and projectile ballistic coefficient to a stable orbit specified in terms of absolute altitude and aircraft airspeed.

PREFACE

The inherent ineffectiveness of tactical dive bombing in Southeast Asia (SEA) against mobile targets on the North Vietnamese resupply route (Ho Chi Minh Trail) led the U.S. Air Force to develop and employ the AC-130 gunship. A leading engineer in the development, test, evaluation, deployment and modification phases of the AC-130 was Dr. Richard E. Willes, now President of Research, Analysis, Development, Inc., of Colorado Springs, Colorado. While in SEA, Dr. Willes solved the problem of determining the nominal firing orbit for a gunship operating in a constant wind and conceived the idea for an implicit fire control system, one in which the fire control problem is solved implicitly by matching the ballistic drop of the projectiles to the change in pylon elevation of the wing, thus avoiding the expense and complexity of sophisticated avionics.

Since August, 1972, the project has been sponsored by the Department of Aeronautics, US Air Force Academy, under the supervision of Lt Col Tom J. Forster. The study was completed in July, 1973. Major contributors to its progress are Major Duane M. Davis and Cadets Glen Strain, Robert Fraser, John Reesee, [illegible]

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

Symbol	Definition
A	Reference area for ballistic drag force
$\underline{a}$	Centripetal acceleration vector
C_D	Ballistic drag coefficient
f	True anomaly
$\underline{g}$	Gravitational acceleration vector
$\underline{H}$	Angular momentum per unit mass
h'	Altitude above mean sea level
L	Lift force
m	Mass
P	Semilatus rectum
P'	Ballistic aerodynamic penetration distance
$\underline{r}$	Position vector
r_b	Ballistic drop
r_c	Radius of curvature
$\underline{r_h}$	Vector position in the horizontal plane
r_v	Absolute altitude of orbit above target
r_v'	Distance of pylon point below plane of orbit
$\underline{V}$	Vector velocity
$\underline{V_a}$	Vector true airspeed
V_m	Muzzle velocity
$\underline{V_w}$	Wind vector

LIST OF SYMBOLS
(Continued)

Symbol	Definition
β'	Inverse atmospheric scale height
δ	Angle of line of sight depression
ϵ	Elliptical eccentricity
γ	Ballistic flight path angle
ρ	Air density
ϕ	Bank angle

Section 1

GUNSHIP FIRING ORBIT IN THE PRESENCE OF WIND

Introduction

A gunship flies a closed orbit around a target in the presence of wind as shown in Figure 1-3. The line of sight (LOS), pylon point, gun line, and associated angles are depicted in Figure 1-2. The aircraft altitude and true airspeed are held constant, and the wing is pointed at the pylon point.

The first step in the analysis of the gunship orbit is to describe the orbit analytically. This description was made by Willes and Fraser.[1]

Kinematics of the Firing Orbit

The aircraft velocity with respect to the target is depicted in Figure 1-1.

(1.2.1) $\underline{V} = \underline{V_a} + \underline{V_w}$

where

$$|\underline{V_a}| = \text{constant} = V_a$$

$$V_w = \text{constant} .$$

The position vector is expressed as

(1.2.2) $\underline{r} = \underline{r_v} + \underline{r_h}$

where

$$\underline{r_v} = \text{constant.}$$

As the gunship fires to the side (wing points at the pylon point),

(1.2.3) $\underline{r_h} \cdot \underline{V_a} = 0$

[1] Willes, R. E. and C. C. Fraser. A Gunship Fire Control in the Presence of Wind. Research Tech Note, Department of Aeronautics, USAF Academy, CO, 1972.

and

$$(1.2.4) \qquad \frac{d}{dt}\left(\underline{r_h} \cdot \underline{V_a}\right) = \underline{\dot{r}_h} \cdot \underline{V_a} + \underline{r_h} \cdot \underline{\dot{V}_a} = 0$$

where

$$\underline{\dot{V}_a} = \underline{a}$$

is the acceleration provided by the wing lift in the turn so that $\underline{r_h}$ and $\underline{a}$ are opposed (see Figure 1-2). Then it must follow that

$$(1.2.5) \qquad \underline{r_h} \times \underline{\dot{V}_a} = 0 .$$

The angular momentum per unit mass of the horizontal orbit is given by

$$(1.2.6) \qquad \underline{H} = \underline{r_h} \times \underline{V}$$

which by equation (2. 1 1) becomes

$$(1.2.7) \qquad \underline{H} = \underline{r_h} \times (\underline{V_a} + \underline{V_w})$$

In scalar form, equation (1.2.7) gives

$$(1.2.8) \qquad H = r_h V_a \sin \pi/2 + r_h V_w \sin \beta$$

where β is the angle from $\underline{r_h}$ to $\underline{V_w}$.

The above expression is rearranged to give

$$(1.2.9) \qquad \frac{H}{V_a} = r_h \left(1 + \frac{V_w}{V_a} \sin \beta\right)$$

which is the equation of an ellipse with eccentricity,

$$(1.2.10) \qquad \epsilon = \frac{V_w}{V_a}$$

provided H can be shown to be constant and β is the complement of the true anomaly.

From (1.2.7)

$$\underline{\dot{H}} = \underline{\dot{r}_h} \times \underline{V_a} + \underline{\dot{r}_h} \times \underline{V_w} + \underline{r_h} \times \underline{\dot{V}_a} + \underline{r_h} \times \cancelto{0}{\underline{\dot{V}_w}} \, .$$

But

$$\underline{\dot{r}} = \underline{V}$$

and from equation (1.2.2)

$$\underline{\dot{r}} = \underline{\dot{r}_h} + \cancelto{0}{\underline{\dot{r}_w}} = \underline{V} \, .$$

Hence

$$\underline{\dot{H}} = \cancelto{0}{\underline{V} \times \underline{V}} + \underline{r_h} \times \underline{\dot{V}_a}$$

$$= \underline{r_h} \times \underline{a} = 0$$

as a result of equation (1.2.5). Thus, $\underline{H}$ is constant, and equation (1.2.9) is an ellipse and may be expressed in the usual form.

$$(1.2.11) \qquad r_h = \frac{P}{1 + \epsilon \cos f}$$

where P is the semilatus rectum, ϵ is the eccentricity, and f is the true anomaly measured from the perigee position. From equation (1.2.9)

$$r_h = \frac{H/V_a}{1 + \frac{V_w}{V_a} \sin \beta} \, .$$

It is evident that

$$P = H/V_a$$

$$\epsilon = \frac{V_w}{V_a}$$

and $\sin \beta = \cos f$. This last correspondence requires that the wind vector be positioned perpendicular to the major axis of the ellipse so that β and f are complementary. See Figure 1-3 for the elliptical firing orbit in the presence of a constant wind.

Summary of Orbit Analysis

It has been shown that for an aircraft flying at constant altitude and true airspeed in a constant wind environment, the firing orbit is an ellipse with the target located at a focal point:

$$(1.3.1) \quad r_h = \frac{P}{1 + \epsilon \cos f}$$

where
$$P = H/V_a$$
$$H = |\underline{r_h} \times \underline{V}|$$
$$\epsilon = V_w/V_a$$

and $\underline{V_w}$ is in the direction of the minor axis.

Section 2

VARIATION OF PYLON ALTITUDE

Introduction

The basis of an implicit gunship fire control scheme is first a repeatable closed orbit (determined in Section 1), second a variation in the pylon altitude as the aircraft moves around the orbit, and third the matching of the ballistic drop to the change in pylon altitude.

The work in this section was performed by Willes and Fraser in 1972.[1]

Variation of Pylon Altitude

For constant altitude it is required that

$$L \cos \phi = mg$$

in the horizontal direction

$$L \sin \phi = ma$$

thus

$$(2.2.1) \qquad a = g \tan \phi = \dot{V}_a \; .$$

Pointing the wing at the pylon point over the target requires

$$(2.2.2) \qquad \frac{d}{dt} (\underline{r}_h \cdot \underline{V}_a) = 0$$

which results in

$$r_h \dot{V}_a = V_a^2 (1 + \epsilon \cos f)$$

but

$$\dot{V}_a = a = g \tan \phi$$

so that

[1]Willes, R. E. and C. D. Fraser. A Gunship Fire Control in the Presence of Wind. Research Tech Note, Department of Aeronautics, USAF Academy, CO, 1972.

(2.2.3) $$\tan \phi = \frac{V_a^2 (1 + \epsilon \cos f)}{r_h g}$$

and the pylon altitude is given by

(2.2.4) $$r_v' = r_h \tan \phi = \frac{V_a^2}{g} (1 + \epsilon \cos f)$$

which is seen to vary with the true anomaly according to

(2.2.5) $$\frac{dr_v'}{df} = \frac{-V_a^2}{g} \sin f.$$

This is the rate of change of pylon altitude that will eventually specify the choice of gun for the system.

Section 3

STABILITY OF THE FIRING ORBIT

Introduction

An orbit is considered stable if when the aircraft is perturbed off orbit the act of repositioning the line of sight on target will bring the gunship back into the nominal elliptical orbit. In short, if off orbit by an amount Δr_h with LOS off target as shown in Figure 3.1, and then the LOS is repositioned on target, if the change in radius of curvature is less than Δr_h, the aircraft will rejoin the nominal orbit at a later time. Thus the criterion for stability is

$$\frac{dr_c}{dr_h} < 1 . \tag{3.1.1}$$

Criterion for Orbit Stability

The radius of curvature for an ellipse is given by

$$r_c = \frac{P}{(1 + \epsilon \cos f)^3} \left[1 + 2\epsilon \cos f + \epsilon^2\right]^{3/2} . \tag{3.2.1}$$

The bank angle is given by equation (2.2.3)

$$\tan \phi = \frac{V_a^2}{g\, r_h} (1 + \epsilon \cos f)$$

which is used with (3.2.1) to obtain the change in radius of curvature caused by a change in the angle of bank:

$$\frac{dr_c}{d\phi} = - \frac{V_a^2}{g \sin^2 \phi} \frac{(1 + 2\epsilon \cos f + \epsilon^2)^{3/2}}{1 + \epsilon \cos f} . \tag{3.2.2}$$

From Figure 1.2 it is seen that

$$r_h = r_v \cot (\phi + \delta) . \tag{3.2.2a}$$

Thus

$$(3.2.3) \qquad \frac{dr_h}{d\phi} = - \frac{r_v}{\sin^2(\phi + \delta)} .$$

Combine equations (3.2.2) and (3.2.3) with (2.2.3) and trigonometric identities to obtain

$$(3.2.4) \qquad \frac{dr_c}{dr_h} = \frac{(1 + 2\,\epsilon \cos f + \epsilon^2)^{3/2}}{(1 + \epsilon \cos f)^2} \quad \frac{\sin 2\,(\phi + \delta)}{\sin 2\,\phi} .$$

The criterion for firing orbit stability becomes

$$\sin 2\,(\phi + \delta) < \frac{(1 + 2\,\epsilon \cos f + \epsilon^2)^{3/2}}{(1 + \epsilon \cos f)^2} \sin 2\,\phi$$

which constrains the choice of LOS depression angle, δ, to

$$(3.2.5) \qquad \delta > \tfrac{1}{2} \sin^{-1} \left[\frac{(1 + \epsilon \cos f)^2}{(1 + \epsilon \cos f + \epsilon^2)^{3/2}} \sin(\pi - 2\,\phi) \right] - \phi .$$

Note that for zero wind the orbit is circular and

$$\delta > \pi/2 - 2\,\phi .$$

Discussion of Orbit Characteristics

In Section 1 it was shown that the firing orbit is characterized by constant angular momentum so that the velocity distribution around the orbit is Keplerian. This fact means that the angle of bank must be greatest at perigee where the radius of curvature is least, and it must be least at apogee where radius of curvature is largest.

Gunship pilot experience indicates that angles of bank in excess of 25° or 30° are not suitable for firing, so that $\phi_{max} = 25^\circ$ is an appropriate choice for exploring typical stable orbits. Figures 3-2 through 3-6 depict

the two classes of stable orbits that result from equations (3. 2. 5), (3. 2. 2a). (2. 2. 3) for typical eccentricities. An altitude/airspeed combination chosen so that it does not fall on one of the stable orbits results in instability. In this case the act of repositioning the LOS on target when perturbed will result in driving the gunship further out of orbit.

Section 4

BALLISTIC SOLUTION

Introduction

The change in ballistic drop must be matched to the change in pylon altitude to make the fire control system implicit. First it is necessary to obtain an analytical solution for the ballistic trajectory. R. E. Willes outlined this work, which Major Duane Davis and I performed in 1972. Finally, I completely reworked it as a final check on accuracy.

The Ballistic Equations and Solution

The first order ballistic equations in dimensional form (the prime denotes a dimensional quantity) are

$$(4.2.1)\quad \left\{\begin{aligned} \dot{\underline{r}}' &= \underline{V}' \\ \dot{\underline{V}}' &= -\frac{C_D A'}{2m'\rho'} V'^{2}_{a} \frac{\underline{V}'_a}{V'_a} + \underline{g}' \end{aligned}\right.$$

where the air density is approximated by

$$(4.2.2)\quad \rho' = \rho'_o e^{-\beta h'}$$

$\frac{1}{\beta'}$ ≡ atmospheric scale height

h' ≡ altitude above mean sea level

ρ'_o ≡ sea level density.

Equations (4.2.1) are solved using the method of asymptotic expansions. The full solution procedure is quite lengthy, but straightforward. The steps are outlined below:

1. Write the equations in nondimensional form.

2. Transform from time to range as the independent variable.

3. Introduce series expansions according to

$$(4.2.3)\quad \begin{cases} \underline{r} = \underline{r}^{(o)} + \sum_j \epsilon_j \underline{r}_j^{(1)} + \sum_i \sum_j \epsilon_i \epsilon_j \underline{r}_{ij}^{(2)} + \cdots \\ \underline{V} = \underline{V}^{(o)} + \sum_j \epsilon_j \underline{V}_j^{(1)} + \sum_i \sum_j \epsilon_i \epsilon_j \underline{V}_{ij}^{(2)} + \cdots \\ t = t^{(o)} + \sum_j \epsilon_j t_j^{(1)} + \sum_i \sum_j \epsilon_i \epsilon_j t_{ij}^{(2)} + \cdots \\ \dot{t} = \frac{dt}{dr_1} = V_1^{-1} \end{cases}$$

where the ϵ_i are given by

$$(4.2.4)\quad \begin{cases} \epsilon_1 = \beta P & \text{---- } P \equiv \text{aerodynamic penetration distance} \\ \epsilon_2 = \dfrac{V_{w_o}}{V_o} & \text{---- reference wind/initial projectile velocity} \\ \epsilon_3 = \dfrac{C_{D_h}}{C_{D_o}} \equiv \dfrac{\partial}{\partial h}\left(\dfrac{C_D}{C_{D_o}}\right) & \text{---- fractional change in drag coefficient with altitude} \\ \epsilon_4 = \dfrac{C_{D_V}}{C_{D_o}} \equiv \dfrac{\partial}{\partial V}\left(\dfrac{C_D}{C_{D_o}}\right) & \text{---- fractional change in drag coefficient with velocity} \\ \epsilon_5 = \dfrac{\dfrac{2mg_o}{C_{D_o} A \rho_o}}{V_o^2} & \text{---- ratio of gravitational acceleration to aerodynamic deceleration.} \end{cases}$$

4. Substitute the expansions (4.2.3) into the nondimensional equations:

$$\dot{\underline{r}} = \underline{V}$$

$$\dot{\underline{V}} = -C_D e^{-\epsilon_1 h} V_a^2 \frac{\underline{V}_a}{V_a} + \epsilon_5 \underline{g}$$

and collect terms in powers of the ϵ_i. A hierarchy of differential of equations are generated as shown below:

(4.2.5) zeroth order ($\epsilon_i^{(o)}$):

$$\begin{cases} \dot{t}^{(o)} = V_1^{(o)^{-1}} \\ \dot{\underline{r}}^{(o)} = \underline{V}^{(o)} V_1^{(o)^{-1}} \\ \dot{\underline{V}}^{(o)} = -(\underline{V}^{(o)} \cdot \underline{V}^{(o)})^{\frac{1}{2}} \underline{V}^{(o)} V_1^{(o)^{-1}} \end{cases}$$

(4.2.6) first order (ϵ_i^{1}):

$$\begin{cases} \dot{t}_1^{(1)} = -V_{11}^{(1)} V_1^{(o)^{-2}} \\ \dot{t}_2^{(1)} = -V_{12}^{(1)} V_1^{(o)^{-2}} \\ \quad\vdots \\ \dot{t}_6^{(1)} = -V_{16}^{(1)} V_1^{(o)^{-2}} \\ \dot{\underline{r}}_1^{(1)} = \underline{V}_1^{(1)} V_1^{(o)^{-1}} - \underline{V}^{(o)} V_1^{(1)} V_1^{(o)^{-2}} \\ \quad\vdots \\ \dot{\underline{r}}_6^{(1)} = \underline{V}_6^{(1)} V_1^{(o)^{-1}} - \underline{V}^{(o)} V_{16}^{(1)} V_1^{(o)^{-2}} \\ \dot{\underline{V}}_1^{(1)} = -\{(\underline{V}^{(o)} \cdot \underline{V}^{(o)})^{\frac{1}{2}} (\underline{V}^{(o)} \cdot \underline{V}_1^{(1)}) \underline{V}^{(o)} V_1^{(o)^{-1}} + (\underline{V}^{(o)} \cdot \underline{V}^{(o)})^{\frac{1}{2}} \\ \qquad [\underline{V}_1^{(1)} V_1^{(o)^{-1}} - \underline{V}^{(o)} V_{11}^{(1)} V_1^{(o)^{-2}} - h\, \underline{V}^{(o)} V_1^{(o)^{-1}}]\} \\ \dot{\underline{V}}_2^{(1)} = -\{(\underline{V}^{(o)} \cdot \underline{V}^{(o)})^{-\frac{1}{2}} \underline{V}^{(o)} V_1^{(o)^{-1}} (\underline{V}^{(o)} \cdot \underline{V}_2^{(1)} + \underline{V}^{(o)} \cdot \underline{V}_w) + \\ \qquad (\underline{V}^{(o)} \cdot \underline{V}^{(o)})^{\frac{1}{2}} [\underline{V}_2^{(1)} V_1^{(o)^{-1}} - \underline{V}^{(o)} V_{12}^{(1)} V_1^{(o)^{-2}} + \underline{V}_w V_1^{(o)^{-1}}]\} \\ \quad\vdots \\ \dot{\underline{V}}_6^{(1)} = -\{(\underline{V}^{(o)} \cdot \underline{V}^{(o)})^{-\frac{1}{2}} (\underline{V}^{(o)} \underline{V}_6^{(1)}) \underline{V}^{(o)} V_1^{(o)^{-1}} + (\underline{V}^{(o)} \cdot \underline{V}^{(o)})^{\frac{1}{2}} \\ \qquad [\underline{V}_6^{(1)} V_1^{(o)^{-1}} - \underline{V}^{(o)} V_{16}^{(1)} V_1^{(o)^{-2}}]\} + \underline{g}\, V_1^{(o)^{-1}} \end{cases}$$

Solve the zeroth order equations for the specified initial conditions. The results are

$$
(4.2.7)\quad \begin{cases}
t^{(o)} = t_o + V_o^{-1}\left[e^{V_o^{(o)} V_{1o}^{-1}(r_1 - r_{1o})} - 1\right] \\
r_1^{(o)} = \text{independent variable} = r_1 \\
r_2^{(o)} = r_{2o} + V_{2o} V_{1o}^{-1}(r_1 - r_{1o}) \\
r_3^{(o)} = r_{3o} + V_{3o} V_{1o}^{-1}(r_1 - r_{1o}) \\
V_1^{(o)} = V_{1o}\, e^{-V_o^{(o)} V_{1o}^{-1}(r_1 - r_{1o})} \\
V_2^{(o)} = V_{2o}\, e^{-V_o^{(o)} V_{1o}^{-1}(r_1 - r_{1o})} \\
V_3^{(o)} = V_{3o}\, e^{-V_o^{(o)} V_{1o}^{-1}(r_1 - r_{1o})}
\end{cases}
$$

where $\quad V_o^{(o)} = (V_{1o}^2 + V_{2o}^2 + V_{3o}^2)^{\frac{1}{2}}$

and

$$
(4.2.8)\quad \begin{pmatrix} t_o \\ r_{2o} \\ r_{3o} \\ V_{1o} \\ V_{2o} \\ V_{3o} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 - \dfrac{V_w{}'}{V_o{}'} \sin f \cos \gamma \\ \dfrac{V_w{}'}{V_o{}'} \sin f \sin \gamma \\ - \dfrac{V_w{}'}{V_o{}'} \cos f \end{pmatrix}
$$

Refer to Figure 4-1 for the coordinate system.

Solve the first order equations, using the zeroth order solution as necessary. The method used for this solution is the transitional matrix method,[2] which is just a method of characteristics formalized for systems with large numbers of state variables. In short, if a vector state is described by

$$\dot{\underline{x}} = f(\underline{x}) + \underline{k}$$

where $\underline{k}$ is the forcing function and the initial state $\underline{x}(o)$ is known, then the solution is

$$\underline{x}(r_1) = \Phi(r_1, r_{10})\, \underline{x}(r_{10}) + \int_{r_{10}}^{r_1} \Phi(r_1, \tau)\, \underline{k}(\tau)\, d\tau$$

where $$\frac{d}{dr_1} \Phi(r_1, r_{10}) = f(r_1)\, \Phi(r_1, r_{10})$$

and $$\Phi(r_{10}, r_{10}) \equiv I \quad \text{---- the identity matrix .}$$

The first order ballistic terms (nondimensional) are:

$$t_1^{(1)} = h_o \left[1 - \tfrac{1}{2}\left(e^{-r_1} + e^{r_1}\right)\right] + \sin\gamma \left[1 - \left(\tfrac{1}{2} r_1 + \tfrac{3}{4}\right) e^{-r_1} - \tfrac{1}{4} e^{r_1}\right]$$

$$t_2^{(1)} = 2\, V_{w_1} \left(r_1 + 1 - e^{r_1}\right)$$

$$(4.2.9) \qquad t_3^{(1)} = -\sin\gamma \left[1 - \left(\tfrac{1}{2} r_1 + \tfrac{3}{4}\right) e^{-r_1} - \tfrac{1}{4} e^{r_1}\right]$$

$$t_4^{(1)} = \tfrac{1}{2}\left(1 - e^{-r_1}\right) - \frac{1}{6}\left(e^{r_1} - e^{-2r_1}\right)$$

$$t_5^{(1)} = g_1 \left[e^{r_1}\left(1 - r_1\right) - 1\right]$$

$$r_{2_1}^{(1)} = 0 \qquad V_{1_1}^{(1)} = (h_o r_1 + \tfrac{1}{2} r_1^2 \sin\gamma)\, e^{-r_1}$$

$$r_{2_2}^{(1)} = \tfrac{1}{2} V_{w_2} r_1^2 \qquad V_{1_2}^{(1)} = 2V_{w_1}(1 - e^{-r_1})$$

$$r_{2_3}^{(1)} = 0 \qquad V_{1_3}^{(1)} = -\tfrac{1}{2} r_1^2 \sin\gamma\, e^{-r_1}$$

$$r_{2_4}^{(1)} = 0 \qquad V_{1_4}^{(1)} = e^{-r_1}(r_1 - 1 + e^{-r_1})$$

$$r_{2_5}^{(1)} = g_2 (e^{r_1} - r_1 - 1) \qquad V_{1_5}^{(1)} = \tfrac{1}{2} g_1 (e^{r_1} - e^{-r_1})$$

$$r_{3_1}^{(1)} = 0 \qquad V_{2_1}^{(1)} = V_{2_3}^{(1)} = V_{2_4}^{(1)} = 0$$

$$r_{3_2}^{(1)} = \tfrac{1}{2} V_{w_3} r_1^2 \qquad V_{2_2}^{(1)} = V_{w_2}(1 - e^{-r_1})$$

$$r_{3_3}^{(1)} = 0 \qquad V_{2_5}^{(1)} = \tfrac{1}{2} g_2 (e^{r_1} - e^{-r_1})$$

$$r_{3_4}^{(1)} = 0 \qquad V_{3_1}^{(1)} = V_{3_3}^{(1)} = V_{3_4}^{(1)} = 0$$

$$r_{3_5}^{(1)} = g_3 (e^{r_1} - r_1 - 1) \qquad V_{3_2}^{(1)} = V_{w_3}(1 - e^{-r_1})$$

$$V_{3_5}^{(1)} = \tfrac{1}{2} g_3 (e^{r_1} - e^{-r_1})\ .$$

Substitute (4.2.7) with initial conditions (4.2.8) and the first order terms (4.2.9) into the basic series expansions, equation (4.2.3), to obtain the ballistic solution to first order in ϵ_i:

$$t = t^{(o)} + \epsilon_1 \{h_o [1 - \tfrac{1}{2}(e^{-r_1} + e^{r_1})] + \sin\gamma\, [1 - (\tfrac{1}{2} r_1 + \tfrac{3}{4})\, e^{r_1} - \tfrac{1}{4} e^{r_1}]\} +$$

$$\epsilon_2 (2V_{w_1})(r_1 + 1 - e^{r_1}) - \epsilon_3 \sin\gamma\, [1 - (\tfrac{1}{2} r_1 + \tfrac{3}{4})\, e^{-r_1} - \tfrac{1}{4} e^{r_1}] +$$

$$\epsilon_4 [\tfrac{1}{2}(1 - e^{-r_1}) - \frac{1}{6} (e^{r_1} - e^{-2r_1})] + \epsilon_5\, g_1 [e^{r_1}(1 - r_1) - 1]$$

$$r_2 = \frac{\epsilon_2}{2} V_{w_2} r_1^2 + \epsilon_5 g_2 (e^{r_1} - r_1 - 1) + r_2^{(o)}$$

$$r_3 = \frac{\epsilon_2}{2} V_{w_3} r_1^2 + \epsilon_5 g_3 (e^{r_1} - r_1 - 1) + r_3^{(o)}$$

$$V_1 = \epsilon_1 e^{-r_1} (h_o r_1 + \tfrac{1}{2} r_1^2 \sin\gamma) + \epsilon_2 2V_{w_1} (1 - e^{-r_1})$$

$$- \epsilon_3 (\tfrac{1}{2}) r_1^2 \sin\gamma\, e^{-r_1} + \epsilon_4 e^{-r_1} (r_1 - 1 + e^{-r_1}) + V_1^{(o)}$$

$$V_2 = \epsilon_2 V_{w_2} (1 - e^{-r_1}) + \epsilon_5 \frac{g_2}{2} (e^{r_1} - e^{-r_1}) + V_2^{(o)}$$

$$V_3 = \epsilon_2 V_{w_3} (1 - e^{-r_1}) + \epsilon_5 \frac{g_3}{2} (e^{r_1} - e^{-r_1}) + V_3^{(o)} .$$

Adequacy of the First Order Solution

To verify that the above first order approximation to the ballistic equations is sufficient for the task at hand, a complete numerical solution of the ballistic equations was made on a digital computer and the two solutions were compared for flight times typical of gunship ballistic trajectories. The results were very favorable, with the approximate solution providing ninety-five percent accuracy or better in typical wind environments.

The verification of the accuracy of the first order solution was the work of Cadet Glen C. Strain as advised by Major Duane Davis.

Section 5

BALLISTIC MATCHING

Introduction

In March 1973 I outlined the procedure for ballistic matching for a student project which I am presently supervising. It is a tedious task, and it is progressing slowly. The basic procedure is outlined below. Current status of the investigation is also discussed.

Scheme for Matching

Refer to Figure 4.1. It is apparent that the ballistic drop at the firing point is

$$(5.2.1) \qquad r_b = r_s \cos \gamma$$

while from Section 2, equation (2.2.4) gives the pylon altitude as

$$(5.2.2) \qquad r_v' = \frac{V_a^2}{g} (1 + \epsilon \cos f).$$

It is desired to match the change in r_b to the change in r_v' as the gunship flies the elliptical orbit, i.e., as the true anomaly changes. Hence, the matching criterion is

$$(5.2.3) \qquad - \frac{dr_v'}{df} = \frac{dr_b}{df}.$$

Recall that as the bank angle decreases from perigee to apogee, the value of r_v' decreases, and because of the increased slant range the ballistic drop must increase. This fact accounts for the minus sign.

The task at hand is to calculate the derivatives $\frac{dr_b}{df}$ and $\frac{dr_v'}{df}$.

The dimensional solution for r_b is

$$(5.2.4) \quad \frac{V_w^2 \sin f \sin\gamma \cos\gamma}{2 V_m^2 P'} r_1^2 + \left(\frac{P'}{V_m}\right)^2 g \cos\gamma \left(e^{r_1/P'} - \frac{r_1}{P'} - 1\right) = r_b$$

where P' is the aerodynamic penetration and

$$r_1 = [r_h^2 + r_v^2]^{\frac{1}{2}} \cos(\delta - \lambda) = r \cos(\delta - \lambda)$$

(Refer to Figure 4-1).

As the true anomaly varies, the ballistic drop changes according to

$$(5.2.5) \quad \frac{dr_b}{df} = K_1 [r^2 \{\cos f \sin\gamma \cos\gamma + K_2 \sin^2 f (\cos^2\gamma - \sin^2\gamma)\} +$$

$$\frac{2 r_h^2 \epsilon}{1 + \epsilon \cos f} \sin^2 f \cos\gamma] +$$

$$K_3 \left[\frac{r_h^2 \epsilon \sin f}{r(1 + \epsilon \cos f)} \{e^{r \cos(\delta - \lambda)/P'} - 1\}\right]$$

where

$$K_1 = \frac{V_w^2}{V_m^2} \frac{\cos^2(\delta - \lambda)}{2P'}$$

$$K_2 = \frac{-2 V_a^2}{g P} \epsilon \cos^2\phi \quad \text{(P is the semilatus rectum)}$$

$$K_3 = \frac{P'}{V_m^2} g \cos\gamma \cos(\delta - \lambda).$$

It was developed in Section 2 that the pylon altitude changes along the orbit according to

$$(5.2.6) \quad \frac{dr_{v'}}{df} = -\frac{V_a^2}{g} \sin f$$

so that the task of matching ballistic trajectory characteristics to aircraft

orbit is simply that of substituting equations (5.2.5) and (5.2.6) into (5.2.3). Essentially this substituting places a constraint on the gun muzzle velocity, which is

$$(5.2.7) \qquad V_m^2 = \left[\frac{g \epsilon \cos^3 (\delta - \lambda)}{2P'}\right]\left[r^2 \{\cot f \sin \gamma \cos \gamma + K_2 \sin f (\cos^2 \gamma - \sin^2 \gamma)\} + \frac{2 r_h \epsilon \sin f \cos \gamma}{1 + \epsilon \cos f}\right] + \frac{P' g^2 \cos (\delta - \lambda)}{V_a^2} \cos \gamma \left[\frac{r_h \epsilon \sin f}{r (1 + \epsilon \cos f)} (e^{r \cos (\delta - \lambda)/P'} - 1)\right].$$

It is observed that r, γ, K_2 and r_h are all functions of the true anomaly, so that

$$V_m = V_m \{f\} .$$

Thus the matching requirement is for a variable muzzle velocity, which is impractical.

Current Status of Project

It is most likely that the muzzle velocity is a weak function of the true anomaly, and that a mean value can be chosen (for $f = \pi/2$) which will give acceptable firing accuracies at the orbit extremes, perigee, and apogee. The current investigation selects typical stable firing orbit parameters (altitude, true airspeed, wind velocities, and max bank angle) and calculates the corresponding muzzle velocity for several points in the firing orbit. This procedure will identify on-the-shelf weapons that are useable for designated stable orbits and will establish the allowable positions in orbit for firing with a specified accuracy. In short, it will establish the feasibility or infeasibility of the proposed system.

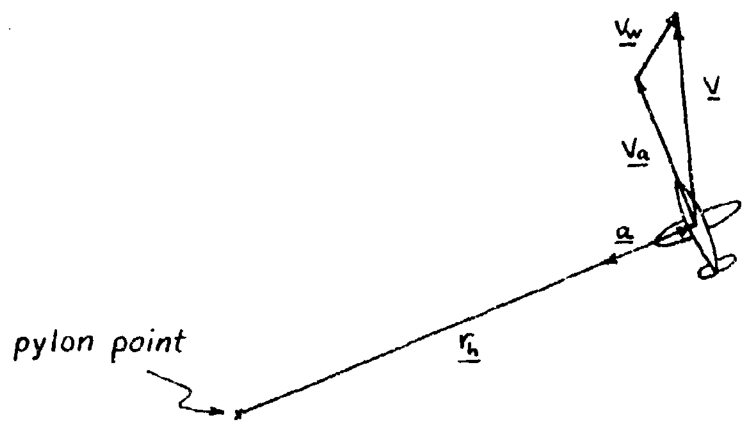

Figure 1. Kinematics of Orbit

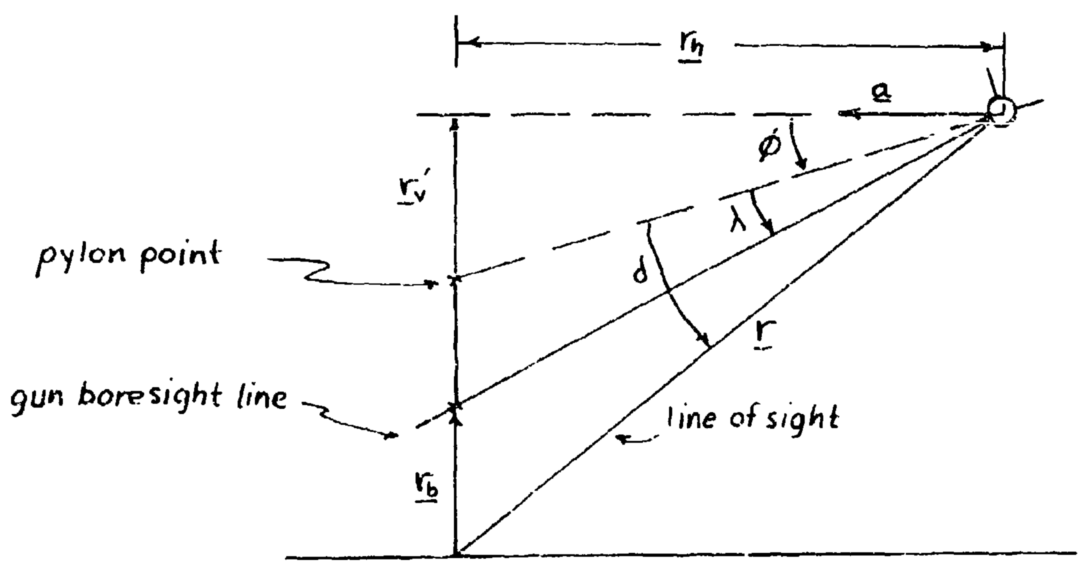

Figure 2. Vector Representation of Aircraft Position

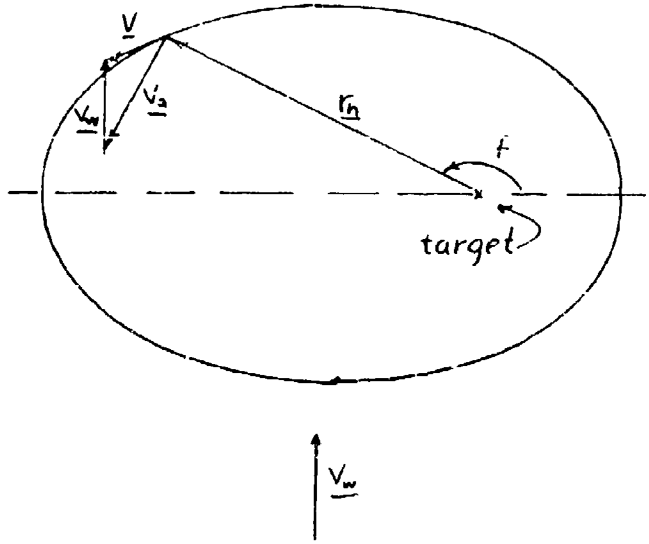

Figure 3. Elliptical Firing Orbit
In the Presence of Wind

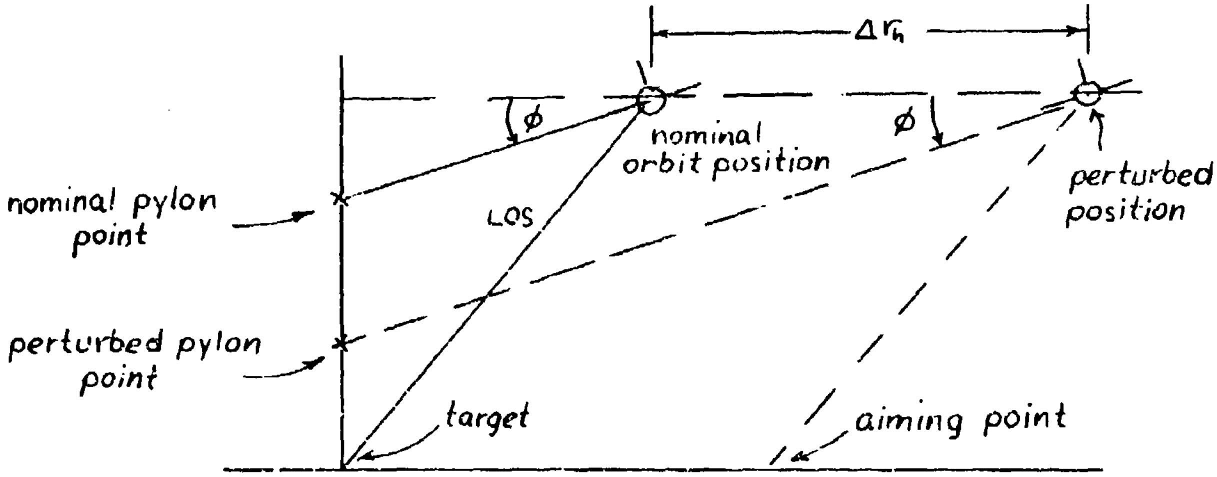

Figure 4. Aircraft Perturbed from Nominal Orbit

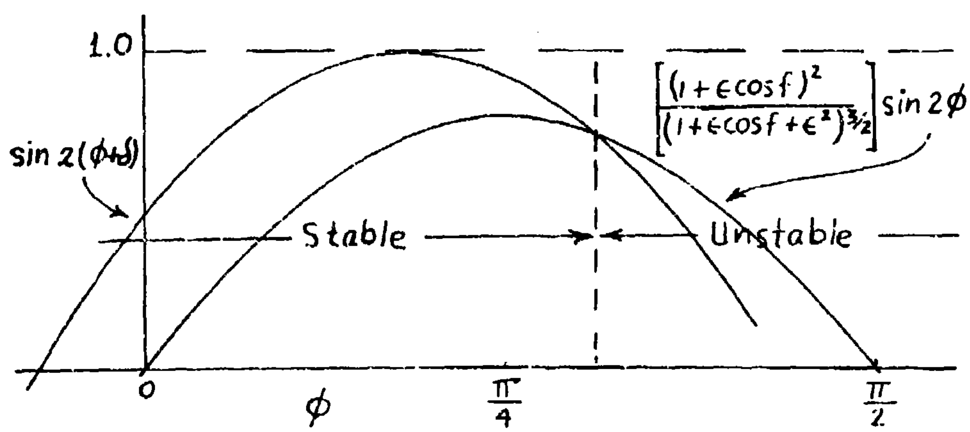

Figure 5. Criterion for Orbit Stability

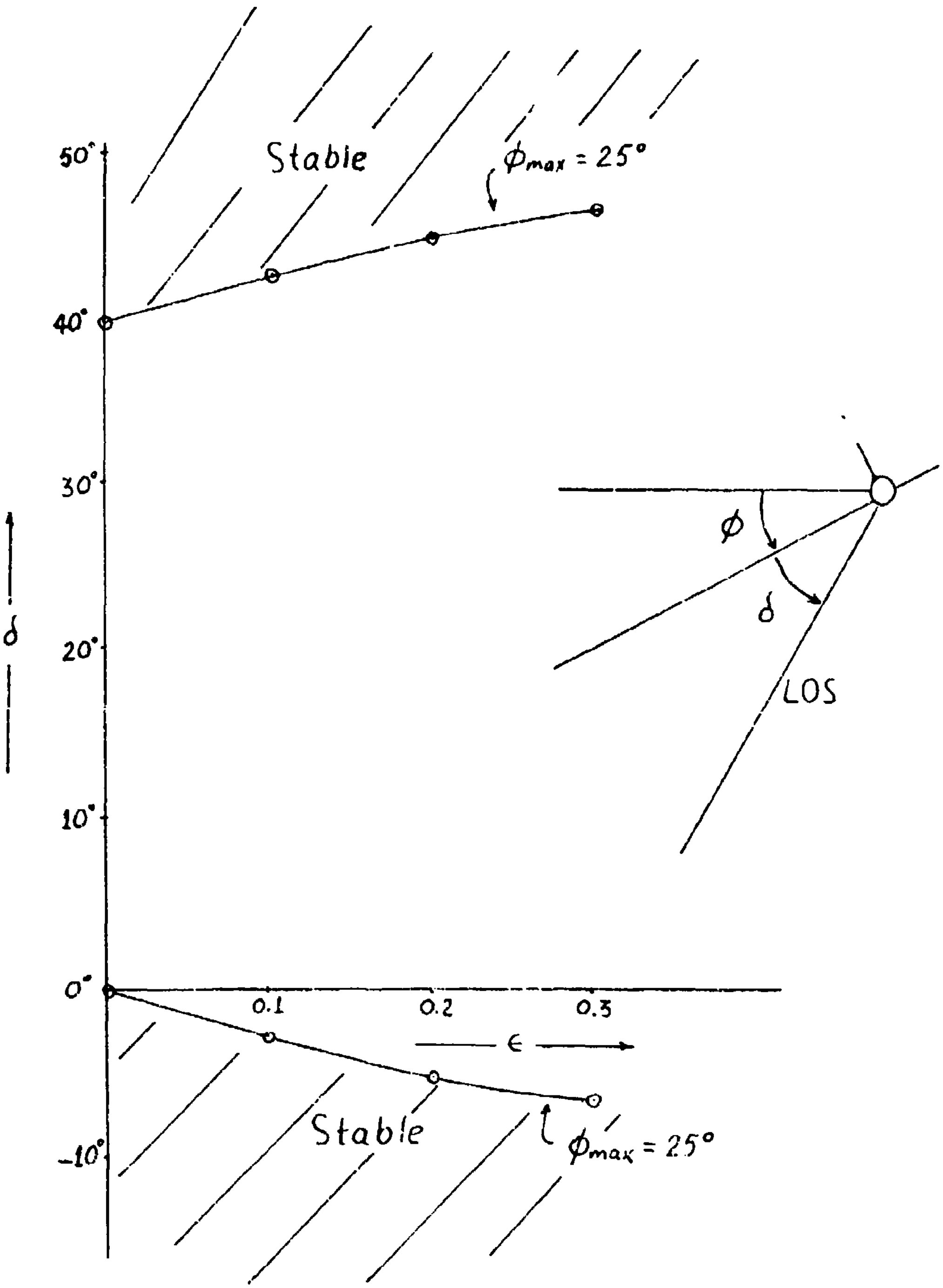

Figure 6. Domains of Stable Sight Depression Angle

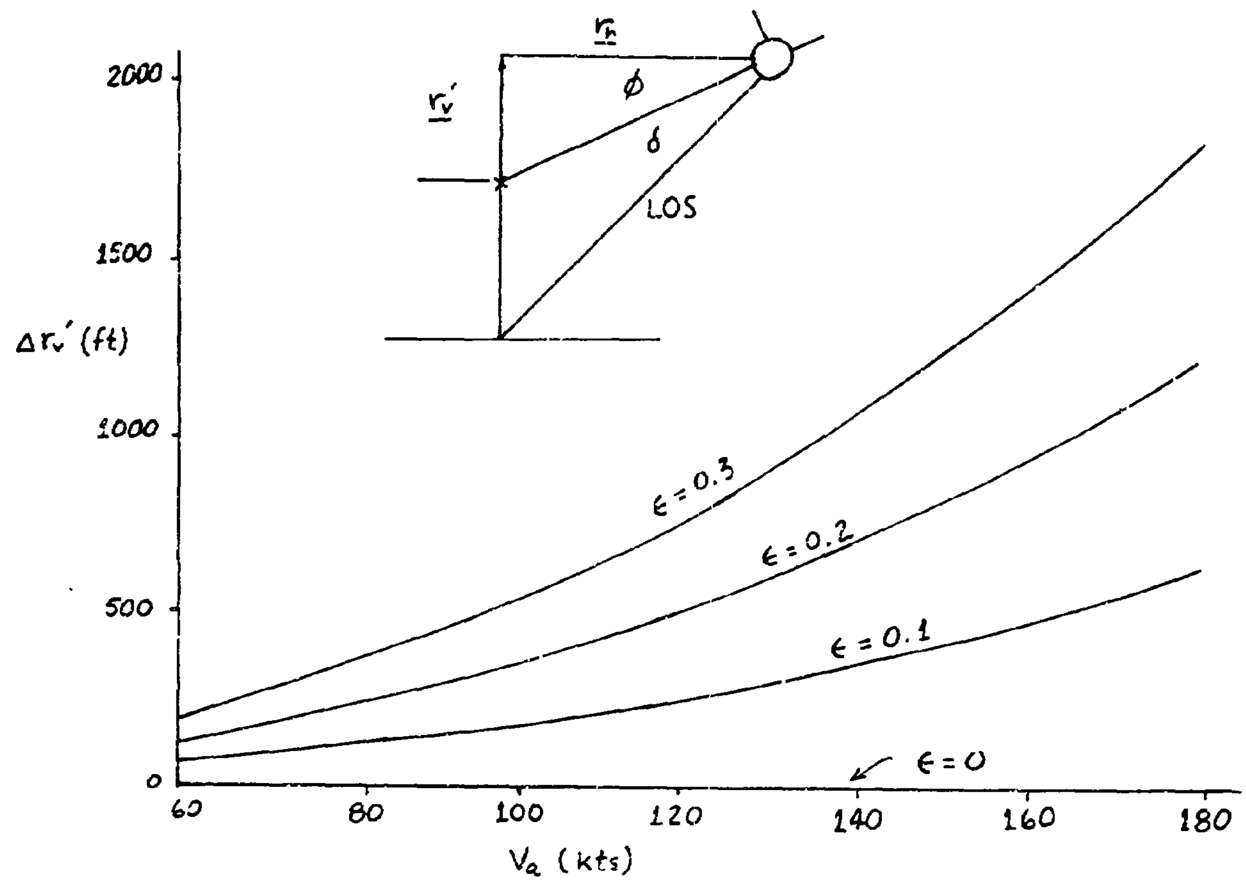

Figure 7. Change In Pylon Altitude from Perigee to Apogee for $\delta = -7^\circ$, $\phi_{max} = 25^\circ$

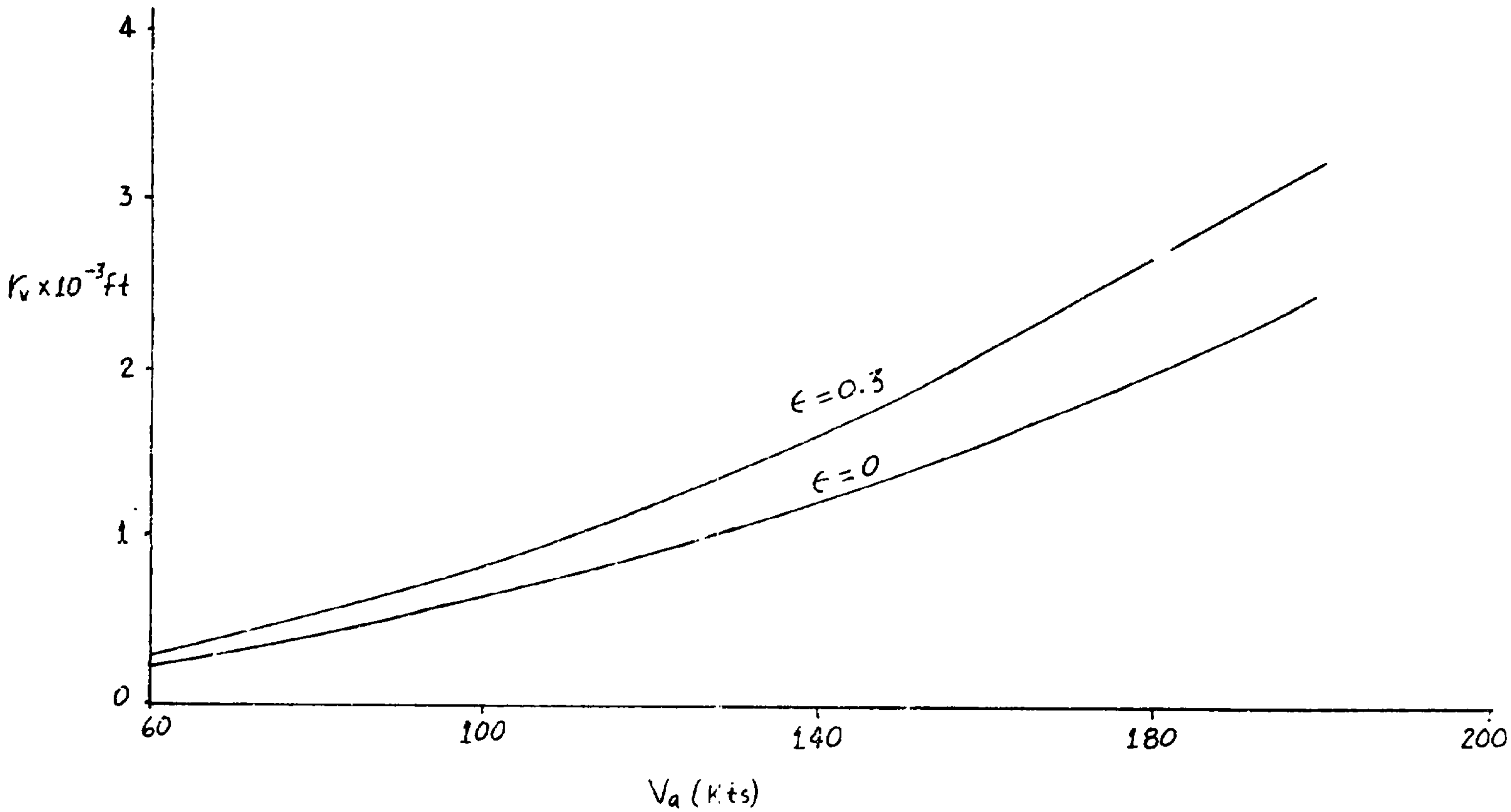

Figure 8. Altitude vs Airspeed for Orbit Stability. $\delta = -7°$, $\phi_{max} = 25°$

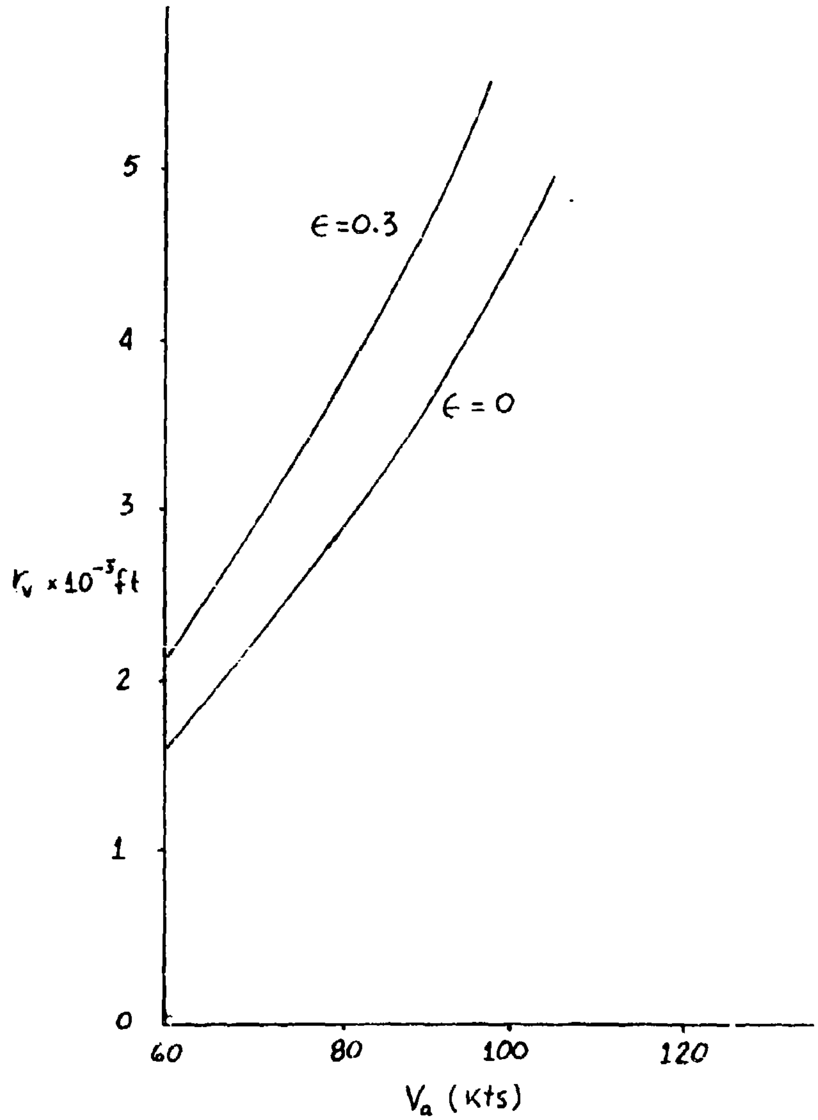

Figure 9. Altitude vs Airspeed for Orbit Stability. $\delta = 47°$, $\phi_{max} = 25°$

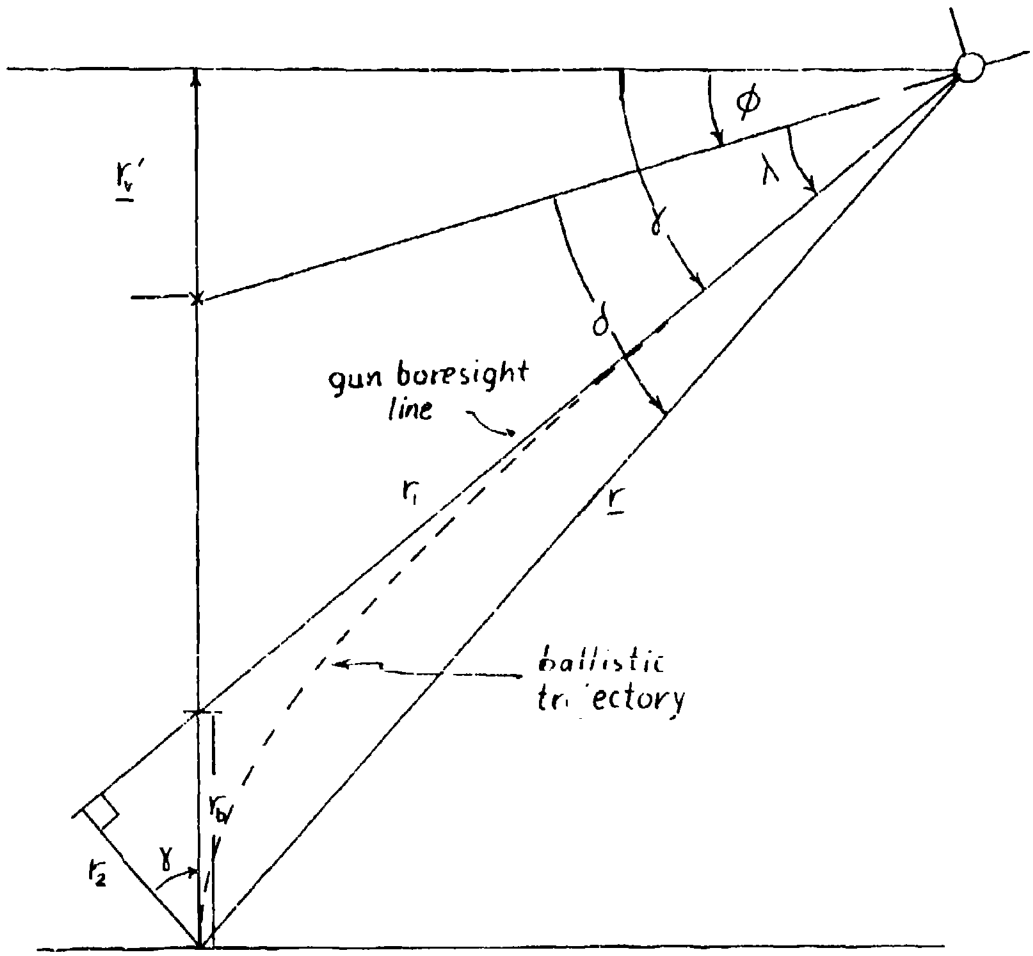

Figure 10. Coordinate System for Ballistic Trajectory

BIBLIOGRAPHY

1. Willes, R. E. and C. D. Fraser. A Gunship Fire Control in the Presence of Wind. Research Tech Note, Department of Aeronautics, USAF Academy, Colorado, 1972.

2. Bryson, A. E., Jr. and Yu-Chi Ho. Applied Optimal Control. Waltham, Massachusetts: Ginn and Company, 1969.